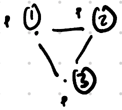
Graphs

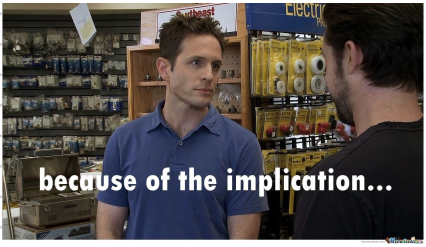


Computer Science Abstraction

[represent objects and relationships]

objects → V

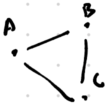
relationships → e edges

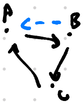

because of the implication...

Always Sunny

Definitions

$\boxed{G} = (V, E)$ — V → set, E → set





$V = \{\ \}$ → set of vertices

$\{1, 2, 3\}$

$\{A, B, C\}$ etc

$E = \{\{\ \}\}$ → multi set

Degree

→ # of incident edges to a vertex v

$d(v) = 4$

$d(A) = \boxed{3}$

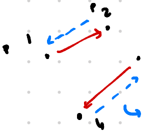
Directed

$\{\overrightarrow{(1,2)}, \overrightarrow{(3,4)}\}$

→ $e = (1,2)$

1 → 2

~~2 → 1~~

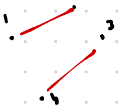


→ not in set

Undirected

$\{\{1,2\} \{3,4\}\}$

→ [1 → 2, 2 → 1]

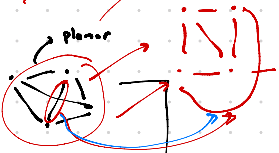


Planarity

→ a graph is planar

→ drawn without edges crossing

→ formally



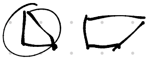
Eulers Formula

→ $\boxed{v} + \boxed{f} = \boxed{e} + 2$

→ $\sum_{i=0}^{f} s_i = 2e$

→ surrounding

→ $s_i = 3$



1
2
3
⋮
1000

$\boxed{f} N \to N$

$6 \quad \mathbb{Z} \to N$

→ $(\exists f)$

s

$\sum_{i=0}^{f} s_i \geq 3f$

→ 2e

→ $[3f \leq 2e]$

[0.11, 0.12, ...]

[1, 2] → 0.115

→

Common Graphs

→ Complete graph

$G = (V, E)$ → n vertices

$|V| = n$

K_5 etc → all possible

K_n → n vertices edges

pentagram → K_5

$G = \boxed{K_n}$ → complete

→ [$d(v) = 4 \ \forall v \in V$

→ $K_n \to d(v) = n-1$]

Paths

Cycles

→ walk

tours

→ [euler tours

euler walks]

→ note

Tree

→ min # of v still be connected → ∃ a path from and v_1 to v_2 in V

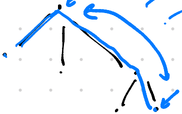
→ $|E| = n-1$ if $|V| = \boxed{n}$ if tree ⇒

→ hypercube

$n = 4$

$|E| = 3$

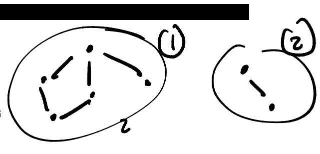
connected (edges)



CS 70 Discrete Mathematics and Probability Theory
Summer 2021

DIS 2A

1 Short Answers - Graphs

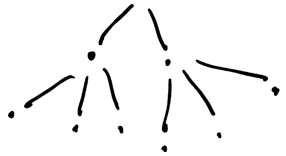


(a) Bob removed a degree 3 node from an n-vertex tree. How many connected components are there in the resulting graph? ↳ 3

(b) Given an n-vertex tree, Bob added 10 edges to it and then Alice removed 5 edges. If the resulting graph has 3 connected components, how many edges must be removed in order to remove all cycles from the resulting graph?

deg 2

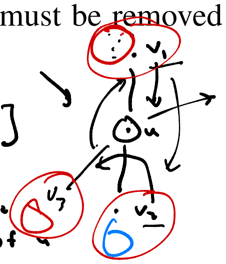
↳ n - vertex graph



[v₁ and v₂] [n - vertex tree = G]

↳ v₁ and v₂

u wlog v₁ and v₂ are neighbors of u



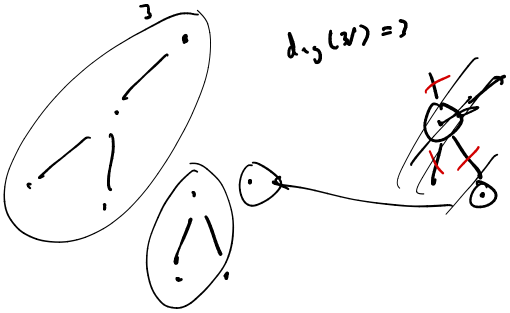
=> v₁ and v₂ are now disconnected because [G is a tree]

=> two groups by def

v₁ → subgraph G₁

v₂ → G₂ G₁ and G₂ connected

↳ => 2 connected comp



n vertex

e = ↳ (n-1)

③ connected components

(n-1)
↳ 2 edges
3 connected

n-1 + 10 - 5 = n + 4

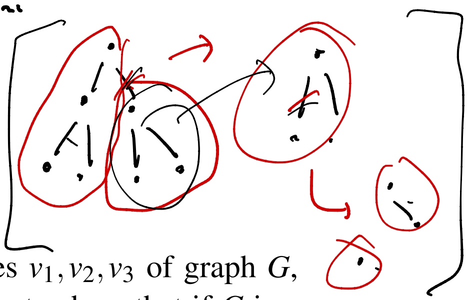
A ↳ A-1, B ↳ B-1, C ↳ C-1

A + B + C = n

n-3 # edges

n+4 → n-3

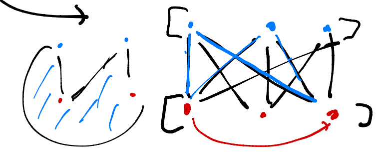
↳ -7



2 Planarity

(a) Prove that $K_{3,3}$ is nonplanar.

(b) Consider graphs with the property T: For every three distinct vertices v_1, v_2, v_3 of graph G, there are at least two edges among them. Use a proof by contradiction to show that if G is a graph on ≥ 7 vertices, and G has property T, then G is nonplanar.



K[3,3]

no edges between red vert and blue vert

Proof contra

↳ Assume K[3,3] planar

euler's formula

↳ (e ≤ 2v - 4) → each face s: ≥ 4

↳ e = 9 v = 6

9 ≤ 12 - 4 → 9 ≤ 8

↳ contra

3 Graph Coloring

Prove that a graph with maximum degree at most k is $(k+1)$-colorable.

Answers

↳ 3
7