Questions Before

Contrapositive

$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$

$P \Rightarrow Q$ ↳ $\wedge$ $\vee$ ¬

$P \Rightarrow Q$

$\neg(P \wedge Q)$ ↳ $\neg P \vee \neg Q$

$P \Rightarrow (\neg R \vee R)$

P Q

3 Number of friends

↳ $n \geq 2$ P ↳ don't invert

at least 2 have some [number of friends]

↳ friendship reciprocated

→ Contradiction

$P \Rightarrow \neg Q$ ↳ impossible

$\neg Q$ → (no same # friends)

↳ $n \geq 2$

[m = # of friendships ↳ $n-1$]

m people at most

1 2 3

Pigeon hole

↳ (bins) balls

↳ $m < n$

→ {bin both}

$m = n-1$

n people

$\max(n-1) < n$

$\neq$

↳ can't occur

Contraposition
↳ "if P then Q" is ≡ "if not Q then not P"

T (P ① ⇒ Q ②) ≡ (¬Q ⇒ ¬P) this leads to ... falling

P ⇒ Q ↳ F
P ⇒ ¬Q → F
Contradiction
Prove P ⇒ ¬Q
↳ is a contradiction
Prove impossible P⇒Q ≠ F

Take note of teaching ↳ will send a feedback form end week with notes

CS 70 Discrete Mathematics and Probability Theory
Summer 2021 at least → **DIS 1B**

x is at least 5

1 Prove or Disprove

$x \geq 5 \longrightarrow x < 5$

For each of the following, either prove the statement, or disprove by finding a counterexample.

solo → (a) $(\forall n \in \mathbb{N})$ if n is odd then $n^2 + 4n$ is odd. → Direct Proof

with me → (b) $(\forall a, b \in \mathbb{R})$ if $a + b \leq 15$ then $(a \leq 11$ or $b \leq 4)$. → contraposition

$\neg$ $Q_1 \lor Q_2$; $\neg Q \Rightarrow \neg P$

group → (c) $(\forall r \in \mathbb{R})$ if r^2 is irrational, then r is irrational. → $\neg Q_1 \land \neg Q_2 \Rightarrow a+b > 15$

$a > 11 \land a > 4$ ↳ $a+b > 15$ ↳ not inclusive $\cancel{\geq}$

(d) $(\forall n \in \mathbb{Z}^+)$ $5n^3 > n!$ (Note: $\mathbb{Z}^+$ is the set of positive integers)

False → $5n^3 < n!$

2 Pigeonhole Principle growth

Prove the following statement: If you put $n+1$ balls into n bins, however you want, then at least one bin must contain at least two balls. This is known as the *pigeonhole principle*.

Contradiction → P ⇒ Q (P = T Q = F)

P: n+1 balls → n bins

¬Q: Suppose all bins have at most one ball ↳ max n balls
n+1 balls ⇒ contradiction

3 Numbers of Friends

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.
(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

$\forall n \in \mathbb{N}$ if n is odd then n^2+4n is odd
P Q

Answers

a) Direct

n odd

$n = 2k+1 \to k \in \mathbb{N}$

n^2+4n ; $n = 2k+1$

$\Rightarrow 4k^2+12k+5$; j

$\to 2(2k^2+6k+2)+1$

odd

$2k+1$

$j \in \mathbb{N} \Rightarrow n^2+4n$ is $2j+1$ odd

sorry for j

d) False oof $(\forall n)$ $+\mathbb{Z}$

by counter

$n=7$ $5 \cdot 7^3 = 1715$

$7! = 5040$

$1715 < 5040$

$5 \cdot n^3$ is $> n!$

if $a+b \le 15$ then $(a \le 11) \vee (b \le 4)$

b) True

Contraposition

$\to \neg Q \Rightarrow \neg P$

$\forall r$

$P \Rightarrow Q$

c) [if r^2 is irrational then r is irrational]

P

$\neg Q \Rightarrow \neg P$

$P' \Rightarrow Q'$

r rational $\frac{a}{b}$ $a, b \in \mathbb{Z}$

$b \neq 0$

r^2 rational $\frac{a^2}{b^2}$ $a, b \in \mathbb{Z}$

$b \neq 0$

$\Rightarrow$ holds

$[a \in \mathbb{Z} \Rightarrow a^2 \in \mathbb{Z}]$

both integers

Direct

r^2 is irrational $\Rightarrow$ r is irrational

r^2 irrational

$\to$ (cannot be $\frac{a}{b}$ $a, b \in \mathbb{Z}$, $b \neq 0$)

r r^2 $r = \sqrt{r^2}$ $\frac{a}{b}$ cut

irrational

$\sqrt{\frac{a}{b}}$ not this

$\sqrt{a}$ $a \in \mathbb{Z}$

Prove

2 Pigeon hole Principle [if $n+1$ items and n containers then 1 container has at least 2 balls]

by contradiction

assume $n+1$ items and n containers

then assume $\neg Q$ all containers have at most 1 ball

max number of balls n

$n \neq n+1$ Contradiction