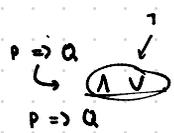


Contrapositive

$\hookrightarrow P \Rightarrow Q$
 \equiv
 $\neg Q \Rightarrow \neg P$

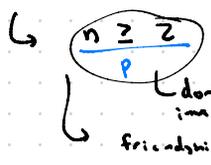


$\neg(P \wedge Q)$
 $\hookrightarrow \neg P \vee \neg Q$

$P \Rightarrow (\neg R \vee R)$

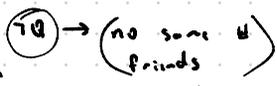
P Q

3 Number of friends



at least 2
 have some number
 of friends

Contradiction
 $P \Rightarrow \neg Q$



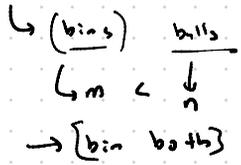
$n \geq 2$

$m = \# \text{ of friendships}$
 $n-1$

m people at most



Pigeon hole



$m = n-1$
 n people
 $\max(n-1) < n$
 \neq
 \hookrightarrow cont occur

$\forall n \in \mathbb{N}$ if n is odd then $n^2 + 4n$ is odd
 a) Direct
 n odd
 $n = 2k+1$
 $n^2 + 4n$; $n = 2k+1$
 $\rightarrow 4k^2 + 12k + 5$
 $\rightarrow 2(2k^2 + 6k + 2) + 1$
 $\hookrightarrow i \in \mathbb{N} \Rightarrow n^2 + 4n$ is $(2j+1)$ odd
 sorry for j

d) False of $(\forall n)$
 by counter
 $(n=7)$
 $5 \cdot 7^2 = 1715$
 $7! = 5040$
 $1715 < 5040$
 $5 \cdot n^2$ is $> n!$

if $a+b \leq 15$ then $(a \leq 11) \vee (b \leq 4)$

b) True
 Contraposition
 $\rightarrow \neg Q \Rightarrow \neg P$
 $P \Rightarrow Q$

c) [if (r^2) is irrational then r is irrational]
 $\neg Q \Rightarrow \neg P$
 r rational $\rightarrow r^2$ rational
 \Rightarrow holds
 $\frac{a}{b}$ $\frac{a^2}{b^2}$
 $a, b \in \mathbb{Z}$ $b \neq 0$
 $a, b \in \mathbb{Z}$ $b \neq 0$
 $[a \in \mathbb{Z} \Rightarrow a^2 \in \mathbb{Z}]$
 balls integers

Direct
 r^2 is irrational $\Rightarrow r$ is irrational
 r^2 irrational
 \rightarrow cannot be $(\frac{a}{b})$ $a, b \in \mathbb{Z}$ $b \neq 0$
 $r = \sqrt{\frac{a}{b}}$ $\frac{a}{b}$ not irrational
 $\hookrightarrow \sqrt{\frac{a}{b}}$ not this
 $\sqrt{a} \notin \mathbb{Z}$

Prove

2 Pigeonhole Principle [if $n+1$ items and n containers
 \hookrightarrow then 1 container has at least 2 balls]
 \hookrightarrow by contradiction
 assume n items and n containers
 then assume $\neg Q$ all containers have at most 1 ball
 \hookrightarrow max number of balls n
 $n \neq n+1$ Contradiction