CS 70 Summer 2021 TA Abhi Gundrala

Start + Berkeley Time

About Me!

3rd year
CS + Cog Sci → Buddhism Minor
psychadelics + neurotechnology decals
guitar
listening to
→ covet
→ tricot
→ hiatus kaiyote
CS 70 Summer 2020

Section Details

Time → 8 to 9 MTu W Th
Small Group
→ more work time
→ participate! (video / audio) :)
→ questions / comments
solutions
notes: posted on website (email this week)
random selection to answer questions
→ feedback

Tips for Success

→ try not fall behind
→ if you do
show up to disc
→ review notes — notes on notes
→ before lecture
→ start HW early
→ try to do a problem daily

CS 70 quick remarks

→ a new way of thinking
→ dont assume anything
→ decartes → remove all doubt
→ slow down [dont panic]
→ spend the time
important in CS, Cog Sci, Life

Content

↳ Notation

$\forall$ "for all"
ex $((\forall x \in \mathbb{Z})$ st $x^0 = 1$

$\exists$ "there exists"
ex $(\exists x \in \mathbb{Z})$ st $(x^2 = 4)$

$\in$ → in

such that

ORDER MATTERS
$\forall x \exists y \neq \forall y \exists x$

Sets
$\mathbb{Z}$ integers $(-5, -4 \cdots 0, \cdots 4, 5)$
$\mathbb{N}$ natural $(0, 1, 2 \cdots)$
$\mathbb{Q}$ rational $(a/b \quad a, b \in \mathbb{Z} \quad b \neq 0)$
$\mathbb{R}$ real
$\mathbb{C}$ complex

TLDR → Think English

Proposition

$P(x)$ → T or F based on x
↳ can be english etc abstract

$P(x)$ ← $x \in$ [red, green ...)
x → T F

Implication
"if $P(x)$ then $Q(x)$"
$P(x) \Rightarrow Q(x)$ definition

excersise
↳ english

TT

P	Q	[P => Q]
T	T	T
T	F	F
F	T	T
F	F	T

only F

Vacuously true

Truth Table
list out all possible value combinations for each proposition

Proofs Briefly

Direct Proof

↳ logical chains
→ start with known info
↳ givens
$(P \Rightarrow Q)$ (P given)
↳ use to construct new givens
P given means ....
↳ chain up to Q given

CS 70 Discrete Mathematics and Probability Theory
Summer 2021 DIS 1A

1 Implication

$\Rightarrow$ $\forall$ $\exists$

Which of the following implications are always true, regardless of P? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

↳ {red, green ..}

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$. pair

True, T ↳ $\forall (x,y)$ T

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

False

$P = [x = y]$

↳ exercise

assum $P \Rightarrow Q$

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

True

2 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$

$\wedge \to$ and $\vee \to$ or

$T \wedge T \to T$

$T \wedge F \to F$

a)

P	Q	$[P \wedge (Q \vee P)]$	$[P \wedge Q]$
T	T	T	T
T	F	F → T	F →
F	T	F	F
F	F	F	F

↳ $T \wedge$

↳ $\neq$

3 Converse and Contrapositive

Consider the statement "if a natural number is divisible by 4, it is divisible by 2" (use "$x \mid y$" to denote x divides y).

$P \Rightarrow Q$

↳ $x \mid y$

(a) Write the statement in propositional logic. Prove that it is true or give a counterexample.

(b) Write the inverse of the implication in English and in propositional logic. Prove that it is true or give a counterexample. (The inverse of an implication $P \implies Q$ is $\neg P \implies \neg Q$.)

(c) Write the converse of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

$Q \Rightarrow P$

(d) Write the contrapositive of the implication in English and in propositional logic. Prove that it is true or give a counterexample.

$\neg Q \Rightarrow \neg P$

opposite of fraction

a) $(\forall x \in \mathbb{N})\ (4 \mid x \Rightarrow 2 \mid x)$

true — 4 divides x — 2 divides x

Proof $4 \mid x \longrightarrow x = 4k \quad k \in \mathbb{Z}$

$4k = (2 \cdot 2)k = 2(2k) \quad [2k \in \mathbb{Z}]$

$\Rightarrow x = 2j = 2 \mid x$

$j = 2k \quad j \in \mathbb{Z}$

b) inverse $(\forall x \in \mathbb{N})$ $\underset{\neg P}{4 \nmid x} \Rightarrow \underset{\neg Q}{2 \nmid x}$

$x = 2$ counter

False

$\equiv$

c) converse

$\forall x \in \mathbb{N}$ $Q \Rightarrow P$

$2 \mid x \Rightarrow 4 \mid x$

$x = 2$ False

d) contrapositive $\neg Q \Rightarrow \neg P$

true

$P \Rightarrow Q \equiv \neg Q \Rightarrow \neg P$ converse inverse

$P \Leftrightarrow Q$

$\begin{bmatrix} P \Rightarrow Q \\ Q \Rightarrow P \end{bmatrix}$ and

$\begin{bmatrix} P \Leftrightarrow Q \\ \text{iff} \end{bmatrix}$

$(P \Rightarrow Q) \wedge (Q \Rightarrow P) \equiv P \Leftrightarrow Q$

$(\neg Q \Rightarrow \neg P) \wedge (Q \Rightarrow P)$